Plan:

(1) complete the calculation of photon Green function

(2) Rules for generic diagrams

(3) Modified propagator for photons + phonons

(4) Resummation

HW 4: 8.11, 8.12, 8.13, 8.15, 8.22

Last time:

$$G_K^{phonon}(t,t') = -i\langle T a_K(t)\, a_K^+(t')\rangle$$
$$= \sum_{n=0}^{\infty} \frac{(-i)^{n+1}}{n!} \int_{-\infty}^{\infty} dt_1 \ldots dt_n \frac{{}_0\langle T\, \hat{a}_K(t)\, \hat{V}(t_1)\ldots \hat{V}(t_n)\, \hat{a}_K^+(t')\rangle_0}{{}_0\langle S(\infty,-\infty)\rangle_0}$$

$n=0 \rightarrow G_K^{ph(0)}(t,t')$

Remember $V = \int dp\, dq\, M\,(c_p^+ d_{p+q} - d_{p-q}^+ c_p)(a_{-q} - a_q^+)$

$n=1 \Rightarrow \emptyset$

$n=2$ some confusion in applying Wick's theorem. Let's concentrate on numerator and go slowly...

$\langle T\hat{a}_K(t)\hat{V}(t_1)\hat{V}(t_2)\hat{a}_K^+(t')\rangle = \langle \text{bosons}\rangle\langle \text{fermions}\rangle$

Separate into boson + fermion parts using Wick's theorem.

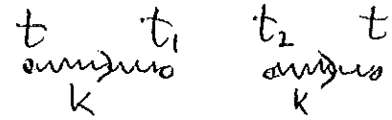
Let's concentrate first on the boson part:

$$\langle \text{bosons}\rangle = \langle T\, \hat{a}_K(t)\,(\hat{a}_{-q_1}(t_1) - \hat{a}_{q_1}^+(t_1))(\hat{a}_{-q_2}(t_2) - \hat{a}_{q_2}^+(t_2))\,\hat{a}_K^+(t')\rangle = \langle T\, \hat{a}_K(t)\, \hat{a}_{-q_1}(t_1)\, \hat{a}_{-q_2}(t_2)\, a_K^+(t')\rangle \Leftarrow \emptyset$$
$$- \langle T\, \hat{a}_K(t)\, \hat{a}_{q_1}^+(t_1)\, \hat{a}_{-q_2}(t_2)\, a_K^+(t')\rangle$$
$$- \langle T\, \hat{a}_K(t)\, \hat{a}_{-q_1}(t_1)\, \hat{a}_{q_2}^+(t_2)\, \hat{a}_K^+(t')\rangle$$
$$+ \langle T\, \hat{a}_K(t)\, \hat{a}_{+q_1}^+(t_1)\, a_{q_2}^+(t_2)\, \hat{a}_K^+(t')\rangle \Leftarrow \emptyset$$

Using Wick's theorem I find four terms $\Rightarrow$ switch to handwritten page 7.

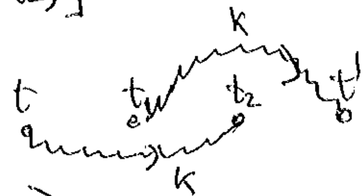
There are four ways to pair up the

**Bosonic part:**



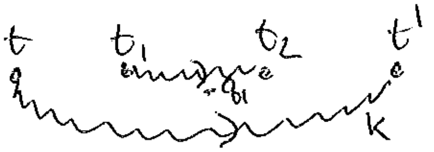

(1) $-\langle T\, \hat{a}_k(t)\, \hat{a}^{+}_{q_1}(t_1)\rangle \langle T\, \hat{a}_{-q_2}(t_2)\, \hat{a}^{+}_{k}(t')\rangle$

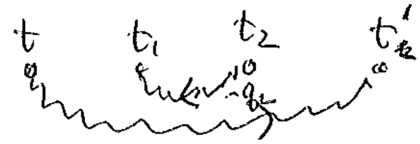
$$= -\left[-i G_k^{ph(0)}(t-t_1)\,\delta(k-q_1)\right]\left[-i G_k^{ph(0)}(t_2-t')\,\delta(k+q_2)\right]$$




(2) $-\langle T\, \hat{a}_k(t)\, \hat{a}^{+}_{q_2}(t_2)\rangle\langle T\, \hat{a}_{-q_1}(t_1)\, \hat{a}^{+}_{k}(t')\rangle$

$$= -\left[-i G_k^{ph(0)}(t-t_2)\,\delta(k-q_2)\right]\left[-i G_k^{ph(0)}(t_1-t')\,\delta(k+q_1)\right]$$

(3) 


(4) 

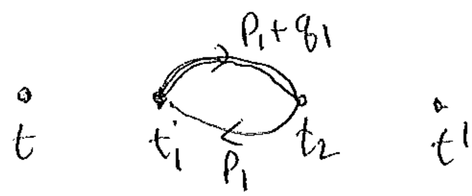

$$G_k^{ph(0)}(t-t')\; G_{-q_1}^{ph(0)}(t_1-t_2)\,\delta(q_1-q_2) \qquad\qquad G_k^{ph(0)}(t-t')\; G_{-q_2}^{ph(0)}(t_2-t_1)\,\delta(-q_2-q_1)$$

**Fermionic Part:** two possible pairings

$$-{}_0\langle T\, \hat{c}^{+}_{p_1}(t_1)\, \hat{d}_{p_1+q_1}(t_1)\, \hat{d}^{+}_{p_2-q_2}(t_2)\, \hat{c}_{p_2}(t_2)\rangle$$

$$= -{}_0\langle T\, \hat{c}^{+}_{p_1}(t_1)\, \hat{c}_{p_2}(t_2)\rangle\langle T\, \hat{d}_{p_1+q_1}(t_1)\, \hat{d}^{+}_{p_2-q_2}(t_2)\rangle$$

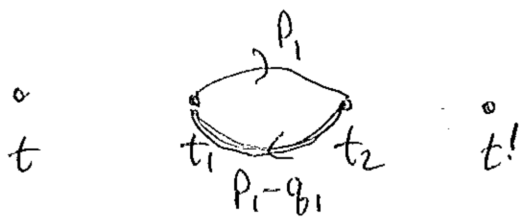
$$= -\left[i G^{(0)}_{v,p_1}(t_2-t_1)\,\delta(p_1-p_2)\right]\left[-i G_{c,p_1+q_1}(t_1-t_2)\,\delta(p_1+q_1-(p_2-q_2))\right]$$




$\Rightarrow$ conduction
— valence

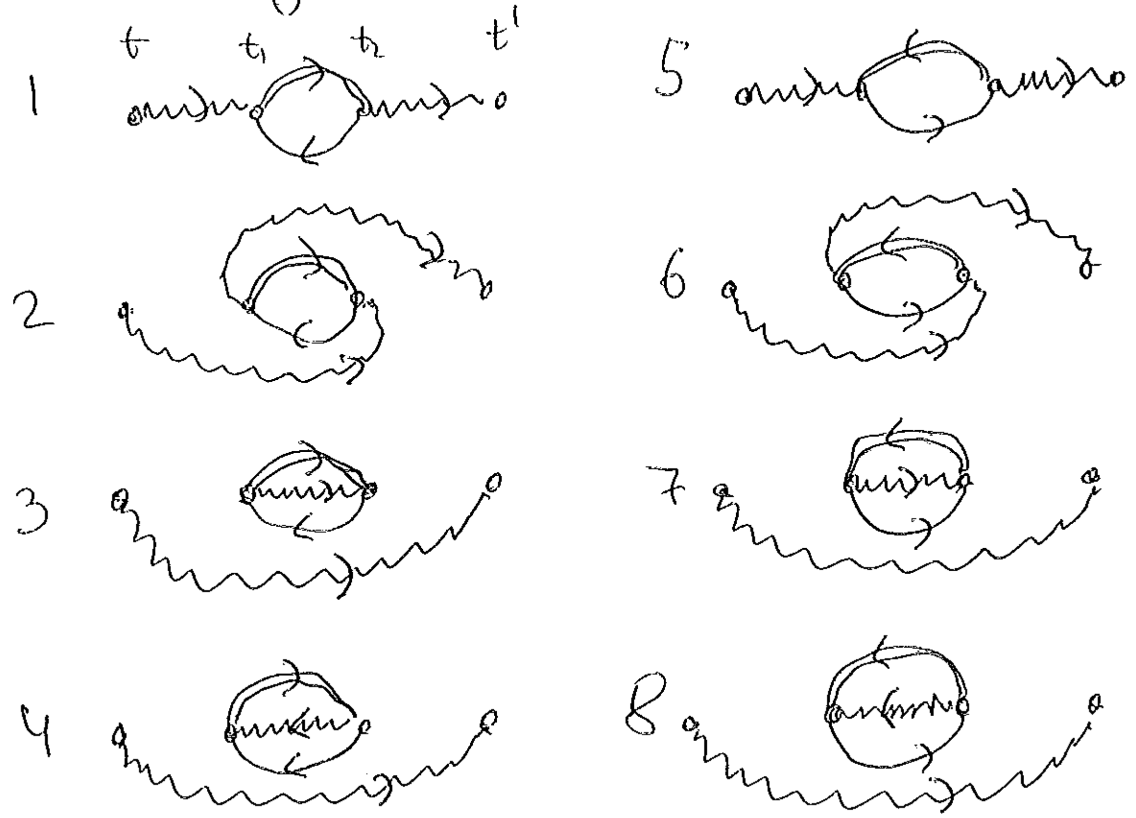
$$-{}_0\langle T\, \hat{d}^{+}_{p_1-q_1}(t_1)\, \hat{d}_{p_2+q_2}(t_2)\rangle_0\langle T\, \hat{c}_{p_1}(t_1)\, \hat{c}^{+}_{p_2}(t_2)\rangle$$

$$= -\left[i G_{c,p_1-q_1}(t_2-t_1)\,\delta(p_1-q_1-(p_2+q_2))\right]\left[-i G_{v,p_1}(t_1-t_2)\,\delta(p_1-p_2)\right]$$



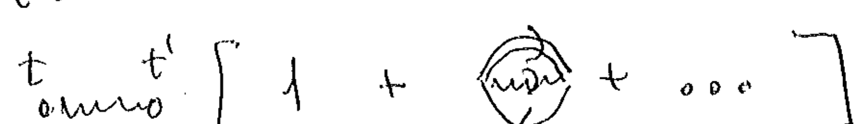

~~Let us concentrate on the fermionic~~

combining all of these we have the following 8 diagrams:




Notes: diagrams 3, 4, 7, 8 are "disconnected"

these diagrams we can write as part of the series




The part in bracket is exactly the denominator $G_k^{\text{Photon}}(t,t')$ expression. full propagator.

This part is called the vacuum polarization and it cancels in the calculation of the photon Green function.

Note 2: $t_1$ + $t_2$ are "dummy" variables

⇒ OK to exchange them

⇒ diagram 1+2 same, 5+6 same } ⇒ this cancels the factor of $\frac{1}{2!}$ in $G^{\text{photon}}$

## Feynman Rules:

Putting those sort of argument together we get the Feynman diagram rules

(1) Draw all distinct + connected diagrams
put arrows on all lines

(2) conserve momentum at each vertex – label all edges with a momentum

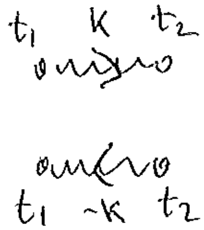
(3) sum over all internal momenta + integrate over all internal times
↳ in Freq. description integrate over all internal freq.

(4) for each line multiply by the Green's function, for each vertex by interaction

(5) ×(−1) for each fermion loop

⇒ see 8.7 of David's book

Note 3: for photons we want to propagate the A-field $\Rightarrow$ we can see this from the structure of vac. polarization, the photon Green functions always come in pairs



$$\Big\} \quad G_k(t_1-t_2) + G_{-k}(t_2-t_1) \equiv \tilde{G}(k, t_1-t_2)$$

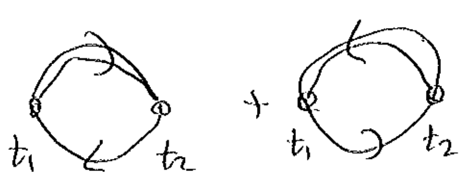
upon fourier transforming, we find

$$\tilde{G}(k,\omega) = G(k,\omega) + G(-k,-\omega) = \frac{1}{\omega-\omega_k+i\epsilon} + \frac{1}{-\omega-\omega_k+i\epsilon}$$

$$= \frac{2\omega_k}{\omega^2-\omega_k^2+i\epsilon} \qquad \text{[D.S. page 396]}$$

where $\omega_k = c|k|$ is the photon dispersion.

## The Fermion Bubble:



$$\equiv \Sigma_{q_1}(t_1,t_2) = \int dp_1\, G^{(0)}_{v,p_1}(t_2-t_1)\, G^{(0)}_{c,p_1+q_1}(t_1-t_2) + G^{(0)}_{c,p_1-q_1}(t_2-t_1)\, G^{(0)}_{v,p_1}(t_1-t_2)$$

Let us fourier transform. Using the fact that the system is time translation invariant we set $t_2=0$.

$$\Sigma_{q_1}(\omega) = \int e^{-i\omega t_1}\,\Sigma(t_1)\,dt_1 = \int dt_1\, e^{-i\omega t_1}\int dp_1 \Big[G^{(0)}_{v,p_1}(-t_1)\, G^{(0)}_{c,p_1+q_1}(t_1) + G^{(0)}_{c,p_1-q_1}(-t_1)\, G^{(0)}_{v,p_1}(t_1)\Big]$$